Examination of Pension Investment Funds in Turkey with Time Series Analysis Methods and Forecasting with ARIMA

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ABSTRACT

In this study, the behavior of private pension investment funds in Turkey, one of the most important investment instruments, was examined using time series analysis methods over a six-year period. Daily price, daily number of shares in circulation, daily number of people, daily total fund value and daily logarithmic return data of selected low, medium and high risk pension investment funds were converted into weekly average data. The movements of the weekly average values of the funds over time were examined graphically using time series analysis methods. The stationarity of the weekly average logarithmic return values of ALZ, AZS and AMZ funds was examined with unit root tests, and the stationarity process was applied to non-stationary returns. Steady weekly average logarithmic return values were modeled with appropriate Autoregressive Integrated Moving Average (ARIMA) models, a one-year forecast was made and compared with the real values. It has been observed that in low risk ALZ funds, forecast values that are closer to reality and have lower errors are obtained with ARIMA methods.

Keywords: *Pension Investment Fund, Risk, Time Series Analysis, ACF, PACF, ADF Test, ARIMA*

1. INTRODUCTION

Most of the retirement plans in the world have a three-pillar structure. The first pillar is the national social security system created by the state, which grants retirement rights to the working individual. The second pillar consists of supplementary retirement funds and plans formed by the employers. The third pillar is private retirement funds established by the private sector, which are voluntary investment and savings systems (Dağlar, 2007).

In Turkey, to complement the public social security system, the Draft Law on the Individual Pension Savings and Investment System was submitted to the Presidency of the Turkish Grand National Assembly on May 16, 2000. The aim was to establish a system based on individual pension accounts to regulate the savings voluntarily made by individuals for retirement (Demirci, 2006).

The "Individual Pension Savings and Investment System Law" No. 4632 was accepted by the Turkish Grand National Assembly on March 28, 2001. The law was published in the Official Gazette No. 24366 on April 7, 2001, and put into practice on October 7, 2001.

The individual pension system is a private retirement system that directs the savings made by individuals during their active working years into long-term investments. It aims to provide additional income when individuals retire, thus enabling them to maintain their living standards (PMC, 2024).

Investment funds are one of the most important elements of the individual pension system. The reason is that the system is built on the basis of amounts accumulated through pension investment funds (Samancı, 2010).

According to the Individual Pension Savings and Investment System Law No. 4632, an individual pension investment fund is an asset designed within the framework of a pension contract by a pension company, where participants' contributions are monitored in individual pension accounts, and managed in accordance with the principles of fiduciary ownership and risk distribution. An individual pension investment fund does not have a legal personality. The individual pension investment fund cannot be used or established for purposes other than those stated in the law in force.

The two main parameters to be considered in individual pension investment funds, as in all investment funds, are the measurement of return and volatility. Return refers to the income obtained from an investment or movable value. Volatility refers to the risk of the change in value of a financial instrument over a certain period of time.

There are many studies on pension investment funds, which have become important investment instruments in financial markets. Studies are generally conducted using the returns of retirement investment funds. Value at Risk (VaR) values of the daily returns of retirement companies were calculated under both the constant variance assumption and conditional heteroscedasticity (Akduğan and Akın, 2013).

Apart from the returns related to pension investment funds, modeling studies are also carried out with price data. The price of five Turkish life, nonlife and pension insurance shares quoted on Istanbul Stock Market (BIST) was estimated via ARIMA models (Kurt and Senel, 2018).

It is realized that various time series analysis methods are used to model time series data observed in a certain period of time for various known investment instruments such as stock market, stocks, foreign currency, gold, dollar, euro and oil, apart from pension investment funds. Stock market, foreign exchange, gold and petroleum returns are predicted with ARIMA, ARCH, GARCH and EGARCH models using Turkish weekly data of BIST 100 (Altuntaş and Çolak, 2015; Değirmenci and Akay, 2017). Holzner et al. (2022) for instance analyzed the impact of public pension expenditures, the assets of pension investment funds, and the benefits paid on macroeconomic volatility.

Results of modelling with ARIMA and deep learning were compared in stock price prediction (Karadağ, 2022). The data of the EREGL share, which is traded in the main metal market on the Borsa Istanbul index, was modeled with ARIMA models and deep learning models using long-short-term memory (LSTM), gated recurrent unit (GRU) and recurrent neural networks (RNN) algorithms (Erden, 2023).

Apart from modeling financial investment instruments, ARIMA models are also preferred in the actuarial literature in modeling time series consisting of premium prices recorded at a certain time. Bortner et al. (2014) compared linear regression and ARIMA in the estimation of calculations related insurance policies. ARIMA models were used in the prediction of life insurance premium production (Çetinkaya, 2019) and in the prediction of fire and natural disaster insurance premiums (Dilmen et al., 2022). Insurance Penetration Rate was modelled via ARIMA models in other studies, as well (Hafiz et al., 2021). Eşsiz and Ordu (2024) employed ARIMA method for the S&P 500 Index basket fund, known for its high-risk, high-return profile among pension mutual funds.

Time series frequently play a crucial role in statistics and economics. A time series consists of a sequence of measurements taken at regular time intervals. This type of analysis is widespread in various scientific fields, but governments commonly use it to forecast economic trends for organizations based on economic data. In addition, time series methods can be used to estimate retirement funds, which are significant financial indicators. Studies have been conducted in which pension investment funds are modeled with artificial neural networks (Onocak and Koç, 2018; Çemrek and Demir, 2021) and ARIMA (Louisa et al., 2022).

In this study, the behaviors of low, medium and high risk private pension investment funds belonging to a private pension company operating in Turkey, which are ALZ, AZS and AMZ, was examined with the help of time series analysis methods over a six-year period. Daily price, daily number of shares in circulation, daily number of people, daily total fund value and daily logarithmic return data of selected low, medium and high risk pension investment funds were converted into weekly average data. The behaviors of the weekly average values of the funds over time were examined graphically using time series analysis methods. The stationarity of the weekly average logarithmic return values of ALZ, AZS and AMZ funds was examined with Augmented Dickey-Fuller (ADF) test, which is one of the unit root tests, and the stationarity process was applied to non-stationary returns. Steady weekly average logarithmic return values were modeled with appropriate ARIMA models, and a one year forecast was performed and compared with real values.

A review of the literature reveals that investment instruments like stocks and gold are typically modeled using ARIMA, while the returns of pension investment funds are often modeled with artificial neural networks. In this study, we analyzed three different pension investment funds with varying risk levels over a 5-year period. This approach is believed to contribute to the literature by providing a comparative analysis of different pension investment funds with distinct risk profiles.

The remainder of the paper is organized as follows. In the Second Section, unit root tests that test stationarity, and ARIMA models, which are linear time series models, will be briefly summarized. In the Third Section, an application will be carried out including time series graphs, ACF and PACF graphs, stationarity tests of some variables related retirement funds such as daily price, daily number of shares in circulation, daily number of people, daily total fund value and daily logarithmic return data and modeling of logarithmic returns of the pension investment fund at three different risk levels with ARIMA. In the last Section, the concluding remarks will be given.

2. METHODOLOGY: TIME SERIES ANALYSIS METHODS

Time series analysis is the examination of data measured over a certain period of time using the mathematical, statistical and econometric methods. Initial analysis can be conducted with time series chart, ACF and PACF charts. The main purpose of time series analysis is to make predictions about the future by using the behaviors of the past data. In most time series methods, stationarity is one of the prerequisites for modeling. After stability is achieved, the modeling and prediction phase can be started.

2.1. Stationarity and Unit Root Test

The concept of stationarity is divided into strictly and weakly stationarity. In strictly stationarity, the distribution function of the series does not change over time. It is very difficult to ensure strictly stationarity in real data applications. In case of weakly stationarity, the expected value ($E(Z_t) = \mu$) and the variance ($V(Z_t) = \sigma^2$) of the Z_t time series are fixed. In addition, its covariance is independent of time ($Cov(Z_t, Z_{t+k}) = \gamma_k$) (Kadılar and Çekim, 2020). What is mentioned with stationarity is generally the concept of weak stationarity, and there is a basic assumption that financial return data is weakly stationary (Tsay, 2005).

One of the most frequently used methods in testing stationarity and determining the degree of difference is unit root tests. The unit root expression is based on testing the hypothesis that the root is equal to the unit value ($\phi_1 = 1$) in the AR(1) model. In the case of unit root, the model is stationary (Eğrioğlu and Baş, 2020).

The most commonly used unit root test is the ADF test (Dickey and Fuller. 1981). The ADF test is based on the reparameterization $\gamma_0 = 1 - \phi_1$. Constant-free and trend-free model ($\Delta x_t = \gamma_0 x_{t-1} + \sum_{k=1}^{m} \gamma_k \Delta x_{t-k} + \varepsilon_t$), the model containing constant term ($\Delta x_t = \beta_0 + \gamma_0 x_{t-1} + \sum_{k=1}^{m} \gamma_k \Delta x_{t-k} + \varepsilon_t$), and the model including the terms constant and trend ($\Delta x_t = \beta_0 + \beta_1 t + \gamma_0 x_{t-1} + \sum_{k=1}^{m} \gamma_k \Delta x_{t-k} + \varepsilon_t$), are defined. In these models, the hypothesis $H_0: \gamma_0 = 0$ is tested (Eğrioğlu and Baş, 2020).

2.2. ARIMA Models

ARIMA models, also known as Box-Jenkins models, are the most basic linear time series models. ARIMA models are the generalization of exponential smoothing methods and are expressed in their most general form as *ARIMA(p,d,q)(P,D,Q)s*, where *p* and *q* are the degrees of auto regression (AR) and moving average (MA) models, respectively. *d* shows the number of differences required for a non-stationary process to become stationary. Box-Jenkins models can be expressed in two different ways as non-seasonal *((ARIMA(p,d,q))* and seasonal *(ARIMA(p,d,q)(P,D,Q)s)* models. In seasonal models, *P* and *Q* shows the degrees of seasonal auto regression (SAR) and seasonal moving average (SMA) models, respectively. *D* shows the number of seasonal differences and s is the period (Kadılar and Çekim, 2020).

The general representation of the $ARIMA(p,d,q)(P,D,Q)_s$ model is given below in Equation (1).

$$\left(1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p\right)\left(1-\Phi_1 B^s-\Phi_2 B^{2s}-\cdots-\Phi_P B^{Ps}\right)(1-B)^d\left(1-B^s\right)^D z_t$$
$$=\left(1-\theta_1 B-\theta_1 B^2-\cdots-\theta_q B^q\right)\left(1-\Theta_1 B^s-\Theta_2 B^{2s}-\cdots-\Theta_Q B^{Qs}\right)\varepsilon_t \qquad (1)$$

In Equation 1, z_t is a stationary time series, ε_t is the error term which is white noise, and the term $(1-B)^d$ shows the closed form of the d-order difference operation. The terms ϕ and θ denote the coefficients of the auto regression and moving average models, respectively, while the terms Φ and Θ are the coefficients of the seasonal models.
In non-seasonal models, *P* and *Q,* which are the degrees of seasonal terms, take the value 0 and Equation 1 turns into Equation 2 as follows.

$$\left(1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p\right)(1-B)^d z_t=\left(1-\theta_1 B-\theta_1 B^2-\cdots-\theta_q B^q\right)\varepsilon_t \qquad (2)$$

ARIMA models are modeled with the a four-stage modeling method called the Box-Jenkins method. These are listed as determining the appropriate model for the time series, estimating the model, diagnostic control and finally the prediction process. According to the ACF and PACF graphs, the models such as auto regression model, moving average model or autoregressive moving average model, which may be suitable for the stationary time series, and its degrees from the significant lag numbers are decided. In the second stage, coefficient significance analysis is performed among possible models. Models whose coefficients are statistically insignificant are eliminated. Then, the diagnostic detection phase is started, which includes joint graphs of the original data and prediction data, confidence interval control, and analysis of whether the errors comply with the white noise process. Finally, among available models, the most suitable model is decided according to the model selection criteria or error criteria. Later predictions are carried out (Kadılar and Çekim, 2020).

3. APPLICATION: EXAMINATION AND FORECASTING OF LOW, MEDIUM AND HIGH RISK PENSION INVESTMENT FUNDS WITH TIME SERIES ANALYSIS METHODS

3.1. Low, Medium and High Risk Pension Investment Funds Data

Pension investment funds in the individual pension system are one of the most crucial components, because the system is built on the principle of accumulated amounts through pension investment funds. An individual pension investment fund comprises investment tools created to manage the contributions of participants within the individual pension accounts under the company's pension contract framework (Elveren, 2002).

According to the Regulation on the Principles Regarding the Establishment and Operations of Pension Investment Funds, when determining the types of funds, attention must be paid to whether the fund's name includes an expression that gives the impression of investment in a particular sector, asset group, or sectors. If such an expression is used, at least 80% of the fund's assets must consist of assets belonging to the sector, asset group, or sectors indicated in the fund's name. Otherwise, this expression cannot be used in the fund's name. Taking these conditions into account, the fund types that guide the practices, which are not restrictive but have a guiding nature, are as follows:

- Income Funds
- Money Market Funds
- Growth Funds
- Fund of Funds
- Contribution Fund
- Precious Metals Funds
- Specialized Funds
- Other Funds

The latest data announced by the Pension Monitoring Center on 04.07.2024 is as follows:

- Participants' Fund Amount: 969.8 billion TL
- State Contribution Fund Amount: 118.5 billion TL
- Total Number of Participants: 9,147,935 people

Private Pension Fund Purchase and Sale Platform (BEFAS) is an electronic platform operated by Takasbank that allows the sale and repurchase of pension investment fund shares by the fund founder pension company to the participants of other pension companies. Participants attending in the private pension system can examine the daily, monthly and annual returns of their funds and obtain information about the risk levels of the funds via BEFAS.

The pension investment fund data were obtained from the websites of the Pension Monitoring Center (EGM) and the Individual Pension Fund Trading Platform (BEFAS). Three pension funds with low, medium, and high risk levels were selected (https://www.egm.org.tr/fonlar/bireysel-emeklilik-fon-alim-satim-platformu-befas/befas/). The codes and descriptions of the funds analyzed in the study are provided below in Table 1

Codes and Names of Low, Medium, and High Risk Pension Investment Funds

Table 1

Fund Code	Fund Name
ALZ	Startup Pension Investment Fund
AZS	Standard Pension Investment Fund
AMZ	Gold Pension Investment Fund

The study utilized daily data on the prices, number of shares in circulation, number of participants, total fund value, and logarithmic returns of the three selected low, medium, and high risk pension investment funds over a six-year period from 2018 to 2023. The daily data were recorded between January 2, 2018, and December 29, 2023, faced issues of missing observations due to weekends, public holidays, and official holidays. Since this issue of missing data arose, the solutions considered were either imputation methods such as replacing missing values with the mean or the previous value to complete the data and work with daily data, or calculating weekly averages to work with weekly data. The estimated missing data must be limited to 20% of the total data. Therefore, the idea of working with daily data was abandoned, and weekly average time series were obtained. Based on the generalization that a year consists of 52 weeks, the data for the 53rd week of 2020, which had 53 weeks, was excluded from the analysis. Missing (empty, 0) or erroneous (negative) observations were corrected by substituting the previous observation or the average values. As a result, a time series of 312 weeks was reached. The dataset includes the following columns for each fund: Date, Price, NumofShare (Number of Shares in Circulation), Numofpeople (Number of Participants), TotalFund (Total Fund Value), and Return (Log Return). Using the Price (P_t) data, logarithmic returns were calculated with the formula $r_t = ln\frac{P_t}{P_{t-1}}$.

The daily data on prices, number of shares in circulation, number of participants, total fund value, and logarithmic returns recorded over a six-year period for low, medium, and high risk pension investment funds were used to examine and interpret the general behaviors of the funds using time series analysis methods. The daily logarithmic return data for all three funds over the first 260 weeks of the five-year period from 2018 to 2022 were used to model and forecast with ARIMA. The last 52 weeks of 2023 were used to test the modeling results.

3.2. Examination of Low, Medium and High Risk Pension Investment Funds with Time Series Analysis Methods

Numerical calculations necessary to examine the behavior of pension investment fund variables over time and forecasting returns using time series methods were performed in the R Studio environment (http://www.rstudio.com/).

Firstly, descriptive statistics of the weekly average daily price, number of shares in circulation, number of participants, total fund value, and logarithmic return variables for low, medium, and high risk pension investment funds covering the period from 2018 to 2023, a 6-year period, were examined and presented in Table 2.

Descriptive Statistics of Price, Number of Share, Number of People, Total Fund and Logarithmic Return for ALZ, AZS and AMZ

Table 2

Statistics for the Price								
	Min.	**1st Qu.**	**Median**	**Mean**	**3rd Qu.**	**Max**	**Variance**	**Std. Dev.**
ALZ	0.01112	0.01446	0.01728	0.01883	0.02231	0.03463	3.31E-05	0.005757
AZS	0.01120	0.01478	0.01865	0.02289	0.02583	0.05444	0.000129	0.011337
AMZ	0.01710	0.02704	0.04853	0.06754	0.10243	0.21212	0.002635	0.051327
Statistics for the Number of Share								
	Min.	**1st Qu.**	**Median**	**Mean**	**3rd Qu.**	**Max**	**Variance**	**Std. Dev.**
ALZ	1.90E+09	2.16E+09	2.50E+09	3.26E+09	4.24E+09	7.29E+09	2.31E+18	1.52E+09
AZS	1.88E+09	2.03E+09	2.30E+09	5.09E+09	8.22E+09	1.13E+10	1.30E+19	3.60E+09
AMZ	1.98E+10	4.38E+10	7.81E+10	7.76E+10	1.08E+11	1.43E+11	1.31E+21	3.6199E+10
Statistics for the Number of People								
	Min.	**1st Qu.**	**Median**	**Mean**	**3rd Qu.**	**Max**	**Variance**	**Std. Dev.**
ALZ	72103	90955	98367	103593	120443	140104	3.02E+08	17372.95
AZS	10460	12398	16180	21218	32435	33850	91399196	9560.293
AMZ	38382	77214	116634	169533	287977	437858	1.4E+10	118258.0
Statistics for the Total Fund								
	Min.	**1st Qu.**	**Median**	**Mean**	**3rd Qu.**	**Max**	**Variance**	**Std. Dev.**
ALZ	26021256	33044634	39161913	67894026	85029172	2.5E+08	2.91E+15	53933074
AZS	24387671	31145908	37246222	1.48E+08	2.52E+08	4.39E+08	2.36E+16	1.54E+08
AMZ	3.38E+08	1.19E+09	3.75E+09	6.92E+09	1.11E+10	3.04E+10	5.73E+19	7.57E+09
Statistics for the Logarithmic Return								
	Min.	**1st Qu.**	**Median**	**Mean**	**3rd Qu.**	**Max**	**Variance**	**Std. Dev.**
ALZ	-0.00002	0.000497	0.00067	0.000771	0.000865	0.004314	2.01E-07	0.000449
AZS	-0.01262	-0.000735	0.000897	0.000935	0.002836	0.011805	1.19E-05	0.003456
AMZ	-0.05492	-0.001636	0.001647	0.001904	0.004675	0.042682	5.24E-05	0.007240

According to Table 2, it is observed that as the risk level increases, the average price, variance, and standard deviation values increase as expected. When the number of shares in circulation is examined, it is seen that high risk pension investment funds have the highest average value. It is noted that ALZ, AZS, and AMZ funds are the most preferred pension investment funds of the company. Following high risk pension investment funds, low risk pension investment funds are preferred. Total fund value is directly related to the number of shares and participants. It is observed that the average total fund value of high risk pension investment funds is higher than the others. Return value, which is one of the most important evaluation criteria for financial investment instruments, is examined as logarithmic return for these three funds. Since variance and standard deviation are the most basic risk measures, as expected, ALZ, being the low risk pension investment fund, has the lowest standard deviation value, while AMZ has the highest standard deviation value. As the risk level increases, the average logarithmic return values also increase. Time series plots ACF and PACF of the variables are also interpreted. ACF and PACF plots are provided in Appendix 1. Logarithmic return values for each fund are modeled using ARIMA models, and the forecasted values are compared with the actual data values. Error values such as MAE, RMSE, MAPE are computed.

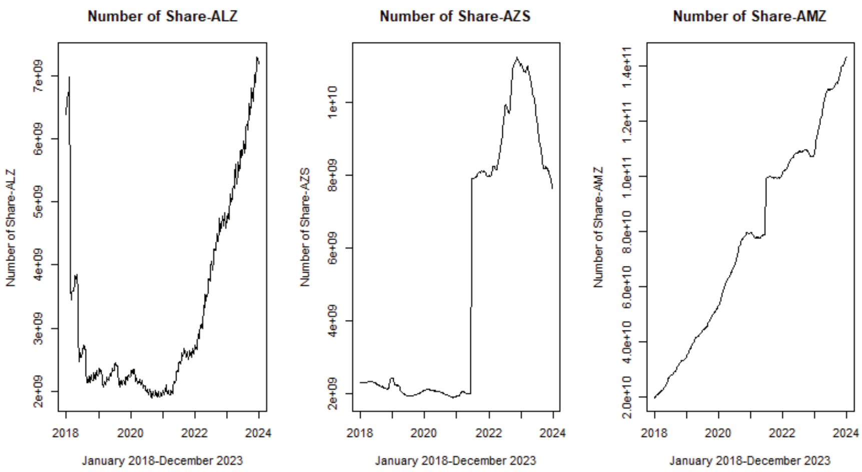
Graphs of Time Series for the Price of ALZ, AZS and AMZ Retirement Funds

Figure 1

The 312-week average daily price time series of low, medium, and high risk pension investment funds were analyzed using time series plots. The price plots of the three pension investment funds are provided in Figure 1. In all three funds, an increasing trend in prices from the first week of 2018 to the last week of 2023 is observed. In the low risk pension investment fund ALZ, almost no random fluctuations have been observed. There is a fairly smooth increasing trend. However, in the medium and high risk funds, random movements are observed along with the increasing trend.

Graphs of Time Series for the Number of Share of ALZ, AZS and AMZ Retirement Funds

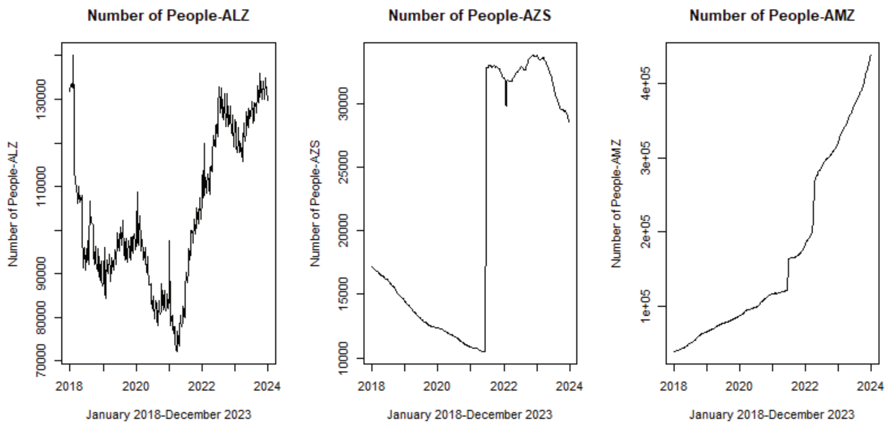
Figure 2



The time series plots of the weekly average number of shares in circulation for low, medium, and high risk pension investment funds over the 6-year period are provided in Figure 2. The number of shares in circulation, the number of participants, and the total fund value are closely related variables. Therefore, it would be beneficial to examine the time series plots of the number of participants provided in Figure 3 and the total fund values provided in Figure 4 together.

Graphs of Time Series for the Number of People of ALZ, AZS and AMZ Retirement Funds

Figure 3

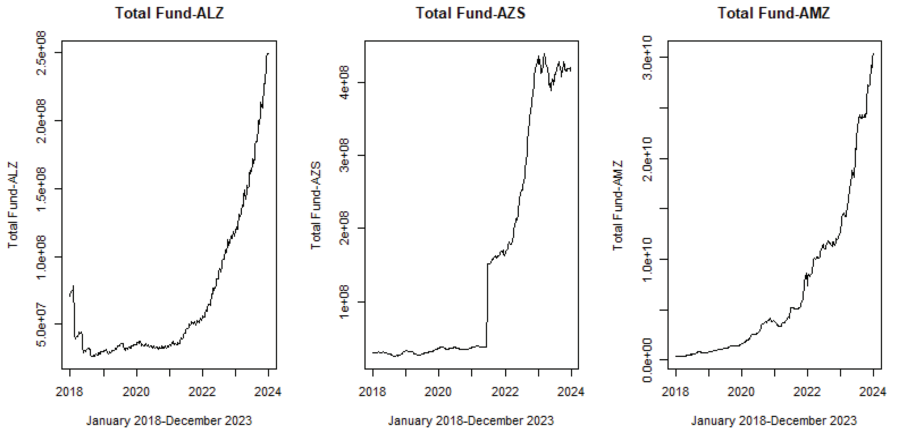


When examining the graphs of the number of shares in circulation and the number of participants for low risk pension investment funds, a sharp decreasing trend until 2019 and a rapidly increasing trend after 2021 are observed. However, the total fund value for low risk pension investment funds decreased between 2018 and 2019, remained stable between 2019 and 2021, and showed an increasing trend after 2021.

The number of shares in circulation for the medium risk pension investment fund AZS remained constant until the middle of 2021. After 2021 it started to increase. As directly related to the number of shares, the number of participants decreased from 2018 to the middle of 2021 and then showed a significant increase from the middle of 2021 onwards. It is observed that the total fund value is closely related to the number of shares in circulation, and their graphs are very similar.

Graphs of Time Series for the Total Fund of ALZ, AZS and AMZ Retirement Funds

Figure 4

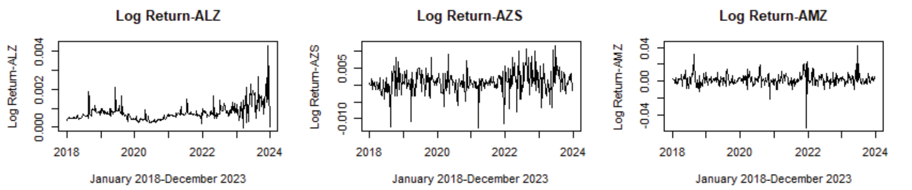


The number of shares in circulation, number of participants, and total fund size for the high risk pension investment fund AMZ shows an increasing trend from 2018 to the end of 2023.

The behaviors of the logarithmic returns for the three funds over the 6-year period are examined using the time series plots. The results are provided in Figure 5.

Graphs of Time Series for the Logarithmic Return of ALZ, AZS and AMZ Retirement Funds

Figure 5



When examining the graphs of the logarithmic return data for the funds, no trend or seasonal fluctuations are observed in any of the three funds. In the low risk pension investment fund ALZ, a slowly increasing trend is realized throughout 2023. However, the logarithmic return data for medium and high risk funds follow a stationary trend. The ACF and PACF plots provided in Appendix 1 for the logarithmic returns also support this stationary result.

3.3. Modeling Weekly Average Logarithmic Returns of Low, Medium, and High Risk Pension Investment Funds with *ARIMA(p,d,q)(P,D,Q)s* Models

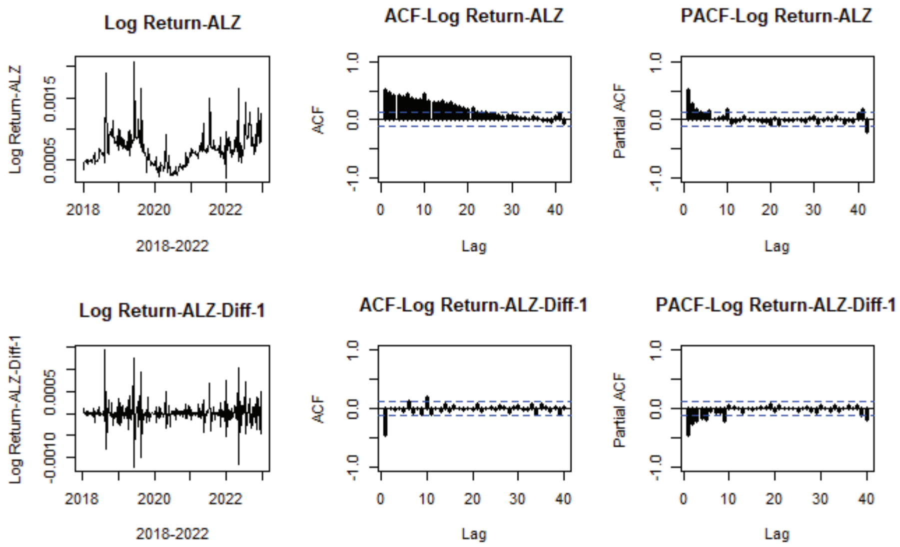
For ARIMA models, the arima() function under the "forecast" package in the R Studio, along with other packages such as "rJava," "XLConnect," "ggplot2," "zoo," "lmtest," and "readxl," are utilized. Weekly average logarithmic returns of low, medium, and high risk pension investment funds have been modeled using time series methods with a total of 260 weeks of data covering the first week of January 2018 to the last week of December 2022. ARIMA models have been used to make 52-week, or one-year, forecasts. The 52 weeks of data observed from the first week of January to the last week of December 2023 have been selected as out-of-sample test data for comparing forecast values to actual data. It should be noted that while 6 years of data were examined in the previous section to analyze the general behaviors of pension investment funds, in this section, only 5 years of data are used for forecasting purposes.

3.3.1. Modeling the Average Logarithmic Return Time Series of the Low Risk ALZ Pension Investment Fund with ARIMA Models

The 260-week time series plot of the logarithmic returns and the differentiated time series plot for the low risk ALZ pension investment fund from 2018 to 2022, along with the ACF and PACF plots, are provided in Figure 6 below. When examining the ACF-PACF plots of the original time series, it is observed that the time series is non-stationary. However, when a difference is taken, the series becomes stationary.

Time Series, ACF, and PACF Plots for the Weekly Average Logarithmic Returns of Original and Differenced Low Risk ALZ Pension Investment Fund for the Period 2018-2022

Figure 6



After graphical examination, the stationarity of the series is examined using the ADF test, one of the most commonly used unit root tests, and the results are provided in Table 3 below.

Stationarity Test Results for the Weekly Average Logarithmic Time Series of ALZ Pension Investment Fund

Table 3

	ADF Unit Root Test - Original Data		
	Draft	**Draft and Trend**	**None**
p-value	< 2.2e-16	< 2.2e-16	1.435e-14
ADF Test Statistics	-5.8643	-5.9322	-1.6269
Adjusted R^2	0.3006	0.3001	0.2143
	ADF Unit Root Test - Differenced Data		
	Draft	**Draft and Trend**	**None**
p-value	< 2.2e-16	< 2.2e-16	< 2.2e-16
ADF Test Statistics	-18.0167	-17.9814	-18.0484
Adjusted R^2	0.7493	0.7463	0.7473
MacKinnon Critical Value (%1)	-3.46	-3.99	-2.58
MacKinnon Critical Value (%5)	-2.88	-3.43	-1.95
MacKinnon Critical Value (%10)	-2.57	-3.13	-1.62

Although the graphical examination of the original series suggests non-stationarity according to the unit root test results provided in Table 3, the time series of the ALZ fund is stationary even without differencing. According to the unit root test results in Table 3, the ADF test statistic values for all models are smaller than the critical values and $p<0.05$, rejecting the hypothesis that there is a unit root in the series. Since the adjusted R^2 values, which indicate goodness of fit, increase after differencing, it is believed that differencing improves the modeling of the series.

Since the ACF plot in Figure 6 appears to decay faster than the PACF plot, it is considered that moving average models might be more appropriate. As the first value in the ACF plot exceeds the confidence limit, models with $q=1$ degrees are established. Within the scope of first-order moving average (MA(1)) models, ARIMA(0,1,1)(0,0,0), ARIMA(0,1,1)(0,0,1)[52], ARIMA(0,1,1)(1,0,0)[52], and ARIMA(0,1,1)(1,0,1)[52] models are established. Although visually, the moving average model seems more suitable for the logarithmic return time series, some first-order autoregressive (AR(1)) models and some autoregressive moving average (ARMA(1,1)) models are also tested. Some of the tested models include ARIMA(1,1,0)(0,0,0)[52], ARIMA(1,1,0)(0,0,1)[52], ARIMA(1,1,0)(1,0,0)[52], ARIMA(1,1,1)(0,0,0)[52], ARIMA(1,1,1)(1,0,0)[52], and ARIMA(1,1,1)(0,0,1)[52]. In some of these models, the coefficients were not statistically significant ($p>0.05$). Among the models with statistically significant coefficients, the ARIMA(0,1,1)(0,0,0) and ARIMA(0,1,1)(10,0)[52] models, which have mismatched real data and forecast data graphs, are eliminated. The adequacy of errors for the last two suitable models is also examined. When the ACF and PACF plots of the errors are compared, it is observed that the lag values of the errors for the ARIMA(1,1,1)(1,0,1)[52] model fall within the confidence limits. The adequacy of errors to the white noise process is tested with the Box-Ljung test, and although the test result does not accept the adequacy to the white noise process, according to the graphical examination, the errors can be considered adequate for the white noise process ($p<0.05$). Additionally, when a comparison is made based on information criteria such as AIC, AICC, and BIC, and error values such as RMSE, MAE, MPE, MAPE, and MASE, it is observed that the ARIMA(1,1,1)(1,0,1)[52] model has the smallest information criteria and error values.

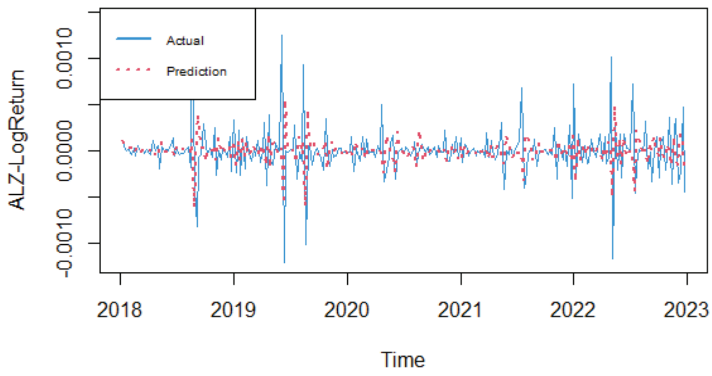
Results of ARIMA Models for Low Risk ALZ Retirement Fund

Table 4

Models	AIC	AICC	BIC	RMSE	MAE	MASE
ARIMA (0,1,1)(0,0,0)	-3509.81	-3509.71	-3499.15	0.0002625092	0.0001473876	0.6746636
ARIMA (0,1,1)(1,0,0)[52]	-3510.66	-3510.50	-3496.45	0.0002605451	0.0001491751	0.6828458
ARIMA (1,1,0)(0,0,1)[52]	-3388.99	-3388.84	-3374.78	0.0003324907	0.0002030711	0.9295536
ARIMA (1,1,1)(1,0,1)[52]	-3567.96	-3567.63	-3546.65	0.0002305417	0.0001345736	0.6160080

The joint plot of ALZ log-return data and the log-return data predicted by the ARIMA(1,1,1)(1,0,1)[52] model

Figure 7



According to Figure 7 and Table 4, among the models tested for the low risk ALZ pension investment fund, the most suitable model is considered to be the ARIMA(1,1,1),(1,0,1)[52] model, and the model results are provided below in Table 5.

Parameter Estimation of ARIMA(1,1,1)(1,0,1)[52] Model for ALZ Retirement Fund

Table 5

Variables	Estimate	Std. Error	z value	Pr(>|z|)
AR (1)	-0.4653	3.0792e-02	-15.1120	< 2.2e-16*
MA (1)	-1.000	9.9765e-03	-100.2334	< 2.2e-16*
SAR (1)	-0.6798	1.4785e-01	-4.5976	4.274e-06*
SMA(1)	0.5686	1.6001e-01	3.5534	0.0003802*

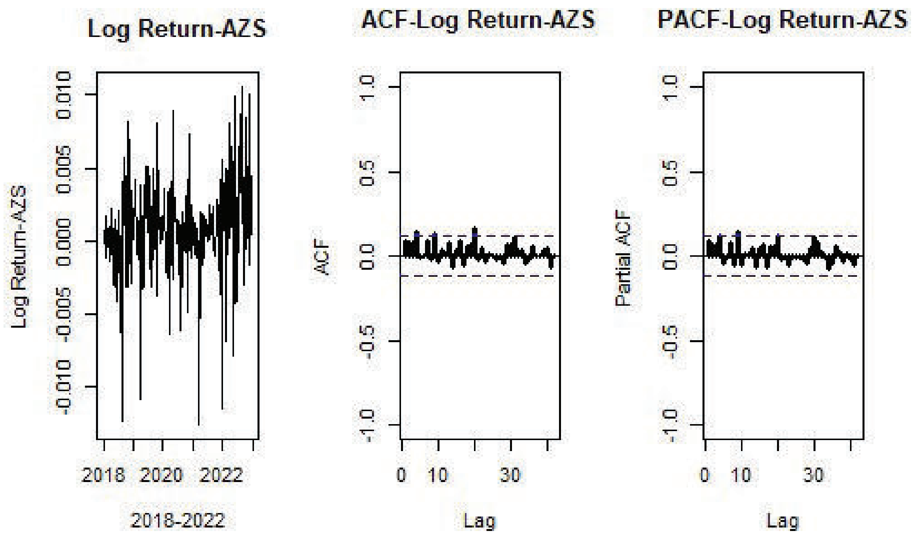
* Statistically significant coefficient (α=0,05)

3.3.2. Modeling the Average Logarithmic Return Time Series of the Medium Risk AZS Pension Investment Fund with ARIMA Models

The graph of the 260-week logarithmic return time series of the medium risk AZS pension investment fund, along with the ACF and PACF plots, is provided in Figure 8 below. Upon examining the ACF and PACF plots of the AZS fund time series, it is observed that this time series is stationary.

Time Series, ACF, and PACF Plots for the Weekly Average Logarithmic Returns of Medium Risk AZS Pension Investment Fund for the Period 2018-2022

Figure 8



The stationarity of the AZS pension investment fund is examined with the ADF test, and the results are presented in Table 5 below. The ADF test results given in Table 6 also support the results obtained in Figure 8.

Stationarity Test Results for the Weekly Average Logarithmic Time Series of AZS Pension Investment Fund

Table 6

	ADF Unit Root Test		
	Draft	**Draft and Trend**	**None**
p-value	< 2.2e-16*	< 2.2e-16*	< 2.2e-16*
ADF Test Statistics	-10.0674	-10.6069	-9.3688
Adjusted R^2	0.4547	0.4698	0.4325
MacKinnon Critical Value (%1)	-3.44	-3.98	-2.58
MacKinnon Critical Value (%5)	-2.87	-3.42	-1.95
MacKinnon Critical Value (%10)	-2.57	-3.13	-1.62

* Statistically significant (α=0,05)

According to the unit root test results shown in Table 6, the AZS pension investment fund is a stationary time series. The ADF test statistics values for all models in Table 6 are smaller than the critical values, and since $p<0.05$, the hypothesis suggesting a unit root in the series is rejected. Although the adjusted R^2 values are not very high, they are acceptable, and modeling can proceed without any adjustments to the data.

AR and MA effects could not be fully determined from the ACF and PACF plots; hence, autoregressive moving average (ARMA) models were primarily tested. In addition to ARMA models, several AR and MA models were also tried. The tested models include ARIMA(1,0,1)(0,0,0), ARIMA(1,0,1)(1,0,1)[52], ARIMA(1,0,1)(1,0,0)[52], ARIMA(1,0,1)(0,0,1)[52], ARIMA(0,1,1)(1,0,0)[52], ARIMA(0,0,1)(0,0,0), ARIMA(0,0,1)(0,0,1)[52], ARIMA(1,0,0)(0,0,1)[52], ARIMA(1,0,0)(0,0,0), ARIMA(1,0,0)(1,0,0)[52], and ARIMA(1,0,0)(0,0,1)[52]. In some of these models, the coefficients were not statistically significant (p > 0.05). Among the models with statistically significant coefficients, ARIMA(1,0,1)(0,0,0) and ARIMA(0,1,1)(1,0,0)[52] showed consistent alignment between real data and forecast data, and their errors seemed to adhere to the ACF-PACF graphs, indicating adherence to a white noise process. The adherence of errors to the white noise process was tested using the Box-Ljung test (p > 0.05). The information criteria such as AIC, AICC, and BIC, as well as error metrics including RMSE, MAE, MPE, MAPE, and MASE, for these two models are presented in Table 7 below. According to Table 7, the ARIMA(1,0,1)(0,0,0) model has lower information criteria values.

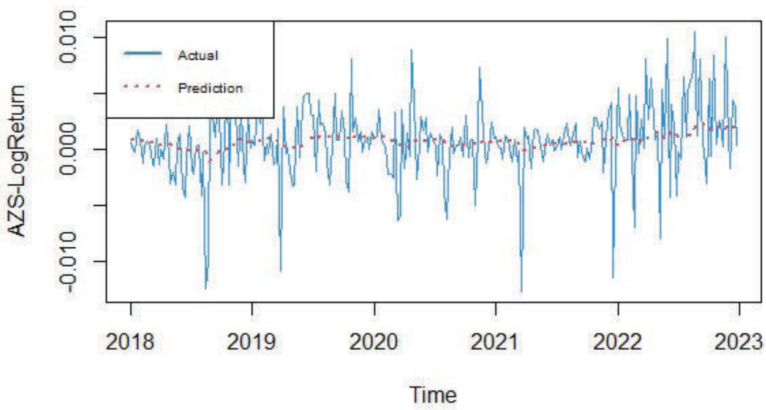
Results of ARIMA Models for Medium Risk AZS Retirement Fund

Table 7

Models	AIC	AICC	BIC	RMSE	MAE	MASE
ARIMA (1,0,1)(0,0,0)	-2219.01	-2218.86	-2204.77	0.003239225	0.002316880	0.7049076
ARIMA (0,1,1)(1,0,0)[52]	-2208.58	-2208.43	-2194.36	0.003327992	0.002318368	0.7053177

The joint plot of AZS log-return data and the log-return data predicted by the ARIMA(1,0,1)(0,0,0) model

Figure 9



According to Table 6 and Figure 9, the most suitable model among the ones tested for the medium risk AZS pension investment fund is accepted to be the ARIMA(1,0,1),(0,0,0) model. The model results are exhibited below in Table 8.

Parameter Estimation of ARIMA(1,0,1)(0,0,0) Model for AZS Retirement Fund

Table 8

Variables	Estimate	Std. Error	*z* value	Pr(>|*z*|)
AR (1)	0.9656	0.05782947	16.6972	< 2.2e-16*
MA (1)	-0.9190	0.08196854	-11.2116	< 2.2e-16*

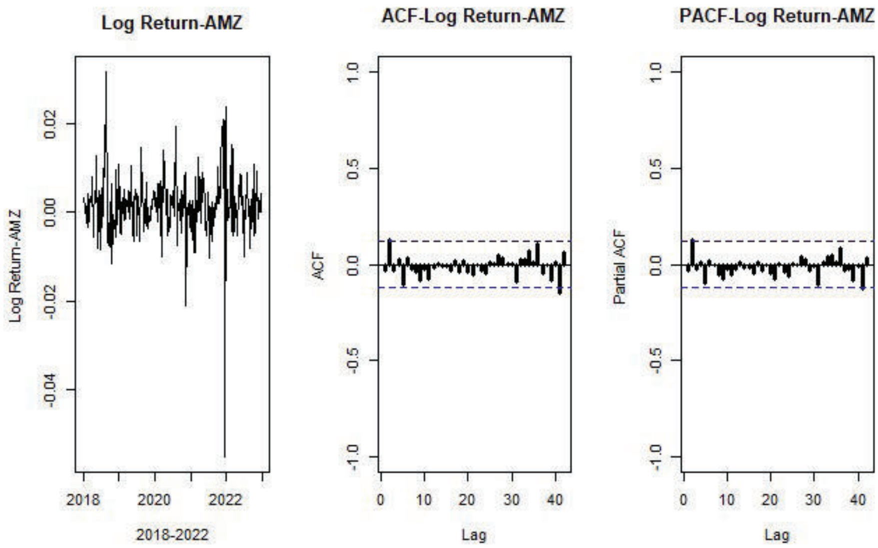
* Statistically significant coefficient (α=0,05)

3.3.3. Modeling the Average Logarithmic Return Time Series of the High Risk AMZ Pension Investment Fund with ARIMA Models

The 260-week logarithmic return time series plot and ACF and PACF plots for the high risk AMZ pension investment fund from 2018 to 2022 are displayed in Figure 10 below. Upon examination of the ACF and PACF plots of the AMZ fund, it is found out that this time series is stationary.

Time Series, ACF, and PACF Plots for the Weekly Average Logarithmic Returns of High Risk AMZ Pension Investment Fund for the Period 2018-2022

Figure 9



The stationarity of the AMZ pension investment fund is examined using the ADF test, and the results are provided in Table 9 below. The ADF test results given in Table 8 also support the results obtained in Figure 9.

Stationarity Test Results for the Weekly Average Logarithmic Time Series of AMZ Pension Investment Fund

Table 9

	ADF Unit Root Test		
	Draft	**Draft and Trend**	**None**
p-value	< 2.2e-16*	< 2.2e-16*	< 2.2e-16*
ADF Test Statistics	-10.0409	-10.0219	-9.3257
Adjusted R^2	0.5235	0.5235	0.5037
MacKinnon Critical Value (%1)	-3.44	-3.98	-2.58
MacKinnon Critical Value (%5)	-2.87	-3.42	-1.95
MacKinnon Critical Value (%10)	-2.57	-3.13	-1.62

* Statistically significant coefficient (α=0,05)

According to the unit root test results given in Table 9, the AMZ pension investment fund exhibits a stationary time series. For all models, the ADF test statistic values are smaller than the critical values, and $p < 0.05$, rejecting the hypothesis of a unit root in the series. The adjusted R^2 values are around 50%, which is an acceptable limit for modeling the data, and modeling can proceed without any adjustments to the data.

Similar to medium risk pension investment funds, ARMA models were primarily attempted due to the inability to accurately determine the AR and MA effects from the ACF and PACF plots. In addition to ARMA models, a few AR and MA models were also tested. Since the 2nd lags are significant in the ACF and PACF plot, 2nd order models will also be attempted. The models tested include ARIMA(1,0,1)(0,0,0), ARIMA(1,0,1)(1,0,1)[52], ARIMA(1,0,1)(1,0,0)[52], ARIMA(1,0,1)(0,0,1)[52], ARIMA(2,0,2)(0,0,0), ARIMA(2,0,2)(1,0,1), ARIMA(2,0,1)(0,0,0), and ARIMA(1,0,2)(0,0,0). In some of these models, all coefficients were not statistically significant, while in others, seasonal coefficients were not statistically significant (p>0.05). Among the models with statistically significant coefficients, the ARIMA(1,0,1)(0,0,0), ARIMA(2,0,2)(1,0,1), and ARIMA(2,0,1)(0,0,0) models exhibited consistent real data and prediction data graphs, and their errors were found to be suitable according to the ACF-PACF plots. The adequacy of errors to the white noise process was tested using the Box-Ljung test (p>0.05). Information criteria such as AIC, AICC, and BIC, as well as error values such as RMSE, MAE, MPE, MAPE, and MASE for these three models, are provided below in Table 10.

Results of ARIMA Models for High Risk AZS Retirement Fund

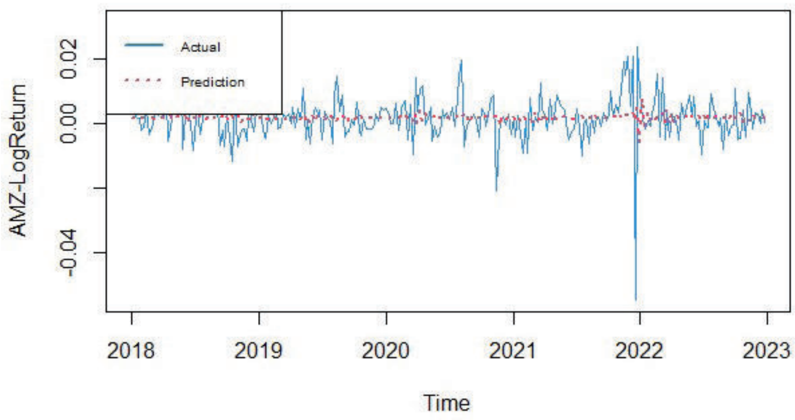
Table 10

Models	AIC	AICC	BIC	RMSE	MAE	MASE
ARIMA (1,0,1)(0,0,0)	-1824.24	-1824.08	-1810.00	0.007106399	0.004668171	0.6700102
ARIMA (2,0,1)(0,0,0)	-1823.66	-1823.43	-1805.86	0.007116759	0.004684180	0.6779852
ARIMA (2,0,2)(1,0,1)[52]	-1818.12	-1817.54	-1789.63	0.007110260	0.004676235	0.6768353

According to Table 10, it is observed that the ARIMA(1,0,1)(0,0,0) model has lower information criteria.

The joint plot of AMZ log-return data and the log-return data predicted by the ARIMA(1,0,1)(0,0,0) model

Figure 10



According to Table 10 and Figure 10, among the models tested for the high risk AMZ pension investment fund, the most suitable model, similar to the medium risk pension fund AZS, is considered to be the first-order autoregressive moving average model, ARIMA(1,0,1),(0,0,0), and the model results are presented below in Table 11.

Parameter Estimation of ARIMA(1,0,1)(0,0,0) Model for AMZ Retirement Fund

Table 11

Variables	Estimate	Std. Error	z value	Pr(>|z|)
AR (1)	-0.8484	0.13372459	-6.3441	2.238e-10 *
MA (1)	0.7856	0.15472554	5.0775	3.824e-07 *

* Statistically significant coefficient (α=0,05)

3.4 Comparison of Forecast Results Obtained with ARIMA Models for Low, Medium, and High Risk Pension Investment Funds

The ARIMA models, as stated in the previous section and detailed in Tables 5, 8, and 11, were used to forecast 52-week (one-year) log returns for low, medium, and high risk pension investment funds. To compare the performance of forecast values, the data from the first 260 weeks were used to determine the models, while the 52-week data observed from the first week of January 2023 to the last week of December was designated as out-of-sample test data to compare forecast values with actual data. Various comparison criteria such as Mean Square Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE) were calculated using the forecast values from the models and the actual values for the three funds. The results are presented below in Table 12.

Comparison Criteria of ARIMA Models for Low, Medium and High Risk Pension Investment Funds

Table 12

	MSE	RMSE	MAE	MAPE
ALZ	2.191588e-06	0.001480401	0.001292598	1.792119
AZS	1.438117e-05	0.003792251	0.003034463	1.266943
AMZ	5.532536e-05	0.007438102	0.004352099	1.842569

When examining the error values, it is observed that forecasts obtained with the ARIMA models determined for all three funds have low MSE values. If we compare across risk levels, it is evident that for low risk pension investment funds, ARIMA models have lower MSE, RMSE, MAE, and MAPE values, indicating that they compute forecast values closer to real data.

4. CONCLUSION

In this study, the behavior of private pension investment funds in Turkey were examined with ARIMA models. For this purpose three private pension investment funds, one low risk (ALZ), one medium risk (AZS) and the other high risk (AMZ), belonging to a private pension company operating in Turkey were analyzed. Weekly return of the data for the time period of 02.01.2018-29.12.2023, covering a 6-year period, was used. The stationarity of the weekly average logarithmic return values of ALZ, AZS, and AMZ funds was assessed using unit root tests, and non-stationary returns were adjusted to achieve stationarity. The stabilized weekly average logarithmic return values were then modeled using suitable ARIMA models. A one-year forecast was generated and compared with actual values.

When the descriptive statistics are examined, it was seen that as the risk level increases, the average logarithmic return values increase as well. It was observed that the average total value of high risk pension investment funds is higher than that of others. The return value, a crucial evaluation criterion for financial investment instruments, was analyzed as the logarithmic return for these three funds.

Upon examining the number of shares in circulation and the number of participants for low risk pension investment funds, a sharp declining trend until 2019 followed by a rapid increase after 2021 was observed. However, the total fund value for low risk pension investment funds decreased between 2018 and 2019, remained stable from 2019 to 2021, and then exhibited an upward trend after 2021.

While analyzing the average logarithmic return time series of the low risk (ALZ) pension investment fund with ARIMA Models, the ACF-PACF plots of the original time series have been examined. It is evident that the series is non-stationary. However, applying differencing to the series renders it stationary. According to modeling results, it is observed that the ARIMA(1,1,1)(1,0,1) model has the smallest information criteria and error values.

When analyzing the average logarithmic return time series of the medium risk (AZS) pension investment fund, it was observed that this time series is stationary. Among the models with statistically significant coefficients, ARIMA(1,0,1)(0,0,0) and ARIMA(0,1,1)(1,0,0)[52] demonstrated consistent alignment between actual data and forecast data. Additionally, their errors appeared to follow the ACF-PACF graphs, suggesting they conformed to a white noise process. Also the ARIMA(1,0,1)(0,0,0) model had lower information criteria values.

When analyzing the average logarithmic return time series of the high risk (AMZ) pension investment fund with ARIMA Models, it was noted that this time series is stationary. The ARIMA(1,0,1)(0,0,0), ARIMA(2,0,2)(1,0,1), and ARIMA(2,0,1)(0,0,0) models exhibited consistent real data and prediction data graphs, and their errors were found to be suitable according to the ACF-PACF plots. Furthermore, ARIMA(1,0,1)(0,0,0) model has lower information criteria.

When examining the error values, it was observed that the forecasts obtained using the ARIMA models for all three funds have low MSE values. Comparing across risk levels, it is evident that for low risk pension investment funds, ARIMA models exhibit lower MSE, RMSE, MAE, and MAPE values, indicating that they generate forecast values closer to the actual data.

When the literature is examined, it is seen that investment instruments such as stocks and gold are generally modeled with ARIMA. The returns of pension investment funds are generally modeled with artificial neural networks. From this perspective, it is thought that this study contributes to the literature in this respect. Eşsiz and Ordu (2024), which is a similar study to this study, worked for 1 fund and a 3-year period. In this study, we studied three different funds with different risk levels and a period of 5 years.

This study is open to improvement in various aspects. This study can be improved by modeling logarithmic returns and by predicting with artificial neural networks and different time series analysis methods such as VAR, ARCH, GARCH models and the results can be compared with ARIMA results.

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Appendix.

A1. ACF and PACF Graphs for the Price, the Number of Share, the Number of People, the Total Fund and the Log-Return of ALZ, AZS and AMZ Retirement Funds