PRECAUTION OF SAVINGS UNDER UNCERTAIN CIRCUMSTANCES

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Abstract

The problem of certainty, of safety which influences the decision-makers' behaviour is both subjective and objective. In a wide temporal perspective with reference to the full horizon of the decision-maker's life, the aspects related to the factors that constrain or relax the decisional behavior gather under the umbrella of the subjective definition of resistance and aversion to fluctuations in consumption, but also to risk, thus becaming part of the decision framework in the decision maker's lifetime.

The ratio between consumption and safe decision-making is a factor that ensures the increase or the decrease in consumption. Under unsafe conditions, the decision-maker will incline towards a saving behaviour, with the view to ensuring a certain continuity in consumption, but also with the benefit of ensuring his own existence. Neither the idea of a cautious behavior, nor that of risky saving is foreign to the decision-maker in such circumstances, that being an issue necessary to define and clarify.

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Introduction

The assumption that consumers have a steady income is obviously unrealistic. Thus, we have to introduce uncertainty in this overview. The consumer should be able to plan knowing that his future earnings from work is subject to changes and they could be higher or lower than expected.

Uncertainty affecting future income brings a new reason to saving. The supposition is that this puts pressure on consumers to increase the accumulation of wealth in order to prepare to face a future risk. This is the so-called precautionary reason for saving and it consists of a cautious consumer behavior.

1. Introductory general notions

Let us consider a simple model with two time periods with a secure income y_0 during a period 0, but an income $\widetilde{y_1}$ in the second period. Suppose that risk is exogenous.

Consumers choose how much to save at the time 0 in order to maximize the utility of they expected all along:

$$\max_c V(s) = u_0(y_0 - s) + Eu_1((1+r)s + \widetilde{y}_1$$

As we can notice, we do not need to suppose at this level is that u_1 equal to u_0. Let us consider $s*$ as optimal saving under uncertain conditions. The first order condition for $s*$ is written as follows:

$$u'_0(y_0 - s^*) = (1+r)Eu'_1((1+r)s^* + \widetilde{y}_1)$$

Note that the desire to save is determined by the marginal utility of consumption expected in the future.

Theoretical cautious consumer behaviour can be derived by comparing $s*$ with savings $\hat{s}$, where $\hat{s}$ a certain insecure future income $\widetilde{y_1}$ is replaced by its expectation:

$$\max_s \hat{V}(s) = u_0(y_0 - s) + u_1((1+r)s + E\widetilde{y}_1)$$

Let us consider $\hat{s}$ as the solution to this maximization program. We want to establish whether optimal savings under uncertain conditions is bigger when uncertainty is removed: $s* > \hat{s}$. Since $\hat{V}$ is concave in s, which is easily verifiable, this happens if and only if $\hat{V}'(s*)$ is negative. This condition implies that by marginal decrease in saving of $s*$, we increase the utility in the long run, under certain conditions. In other words, there will be a demand for cautious savings, if and only if:

$$\begin{aligned}\hat{V}'(s*) &= -u'_0(y_0 - s*) + (1+r)u'_1((1+r)s* + E\widetilde{y}_1)\\ &= (1+r)[u'_1((1+r)s* + E\widetilde{y}_1) - Eu'_1((1+r)s* + \widetilde{y}_1)\\ &\leq 0\end{aligned}$$

where the second equality is achieved by using the max condition. Therefore, the level of savings out of cautious reasons is positive if and only if:

$$Eu'_1((1+r)s* + \widetilde{y}_1) \geq u'_1((1+r)s* + E\widetilde{y}_1)$$

From Jensen’s inequality, this results whenever u'_1 is convex or equivalent, or when u'''_1 is positive. This condition is considered „cautious”. Thus, caution is needed if we want prudent saving to be positive for all possible distributions of the future risk. A consumer with a concave function of marginal utility, on the contrary, will reduce future savings because of the future risk. This individual will manifest what is called „reckless behavior”. Thus, caution corresponds to the third derivative’s positivity of the utility function just as risk aversion is based on the second derivative's negativity. An agent may express an adversary and imprudent behaviour towards risk, for example, by ensuring the risk using an incorrect risk premium and reducing his savings in the case of a future uninsured risk. So, according to definition, a prudent person could be considered a risk-loving person. Yet, there is a link between decreasing risk aversion and caution. As we consider the decreasing risk aversion toward absolute risk (DARA) as a natural assumption, we should consider caution in the same way.

We could measure the intensity of the reason for cautious saving. This can be done by answering the question: what can we consider a safe reduction of future income so as to have the same effect on savings as placing future risk? Where is the symbol of „bonus caution". It is implicitly defined by:

$$Eu'_1(\omega+\bar{y}_1)=u'_1(\omega+E\bar{y}_1-\psi),$$

where w is the wealth accumulated before the second term. This condition means that the desire to save, which is measured by the expected marginal utility of future consumption is not affected by replacing risk with the diminishing expectation of . The caution bonus is seen as positive whenever the agent is prudent. For example, whenever u'''_1 is bigger than 0. It is useful to note at this level of demonstration, that caution bonus is equivalent to the above mentioned risk bonus, but where utility function u_1 would be replaced by the function of marginal utility $-u'_1$. The caution bonus and the risk bonus represent cuts in the welfare, and they have the same effects as the risks added to the expected marginal utility, respectively to EU. This implies that all the results that we previously obtained for risk aversion and risk bonus can be transferred to prudence and caution bonus, by simply substituting u_1 with u'_1. Thus, one can use Arrow-Pratt approximation for the risk bonus in order to obtain an equivalent one for the first caution bonus:

$$\psi \cong \frac{1}{2}P(\omega+E\bar{y}_1)\sigma^2_{\tilde{y}_1}$$

where P represents the degree of absolute caution.

In order to observe to what extent the caution bonus affects savings, let us consider the simple case in which the rate of non-risky savings equals the rate of reduction for time preference, and establish both of them as being equal to 0, ie $r=\delta=0$. In particular, a life-time utility is considered to be $U(c_0,c_1) = u(c_0) + u(c_1)$. We also suppose that $E\tilde{y}_1=y_0$ so that the individual has the same income expected at time 0 and at time 1. Let us suppose, at first, that $\tilde{y}_1$ is non-risky, that meaning that $\tilde{y}_1$ $\tilde{y}_0$. In this formula, the first order condition (6.11) implies that $u'(y_0-s) = u'(y_0+s)$. As already notice, optimal savings is 0, $s^*=0$, since u is strictly concave. In other words, the consumer simply uses his regular income in each period: $c^*_0 = c^*_1 = y_0$.

Now let us suppose that $\tilde{y}_1$ is risky, so the first order condition is:

$$u'(y_0 - s) = Eu'(E\tilde{y}_1 + s) = u'(y_0 + s - \psi)$$

The second equality above derives from our definition of the caution bonus. By solving optimal savings, we obtain $s^*= ½\psi$. Thus, when the consumer is cautious, he will manifest a cautious request for savings, $s^*>0$. Furthermore, an individual who is more prudent will have a bigger caution bonus ψ, in the same way in which an individual who has a greater aversion to risk will have a bigger risk bonus. We conclude that a more cautious consumer will save more than one less prudent. It is very interesting to see that if the happiness function is a quadratic function, which represents a less common assumption in this field, so we have $\psi = 0$, which means the inexistence of a reason for cautious savings.

Literature review

Anghelache and Anghel (2016), Anghelache (2006) describe the statistical tools used to measure the macroeconomic indicators. Anghelache, Manole and Anghel (2016) use the asymptotic normality feature of estimators of singular equation. Anghelache (2016), Dougherty (2007) are concerned with theoretical concepts related to econometric tools, Anghelache, Manole and Anghel (2015) study the tools of economic, financial and banking information models. Anghelache and Sacala(2014) describe the characteristics of the Romanian business environment in terms of capital investment. Bloom (2009) assess the impact of uncertainty, Bloom, Bond and Van Reenen (2007) analyze the impact of uncertainty on investmentS, Bolton, Wang and Yang (2014), Grenadier and Wang (2007) have similar preferences. Hafner and Wallmeier (2008) focuse on volatility as an influencing factor on investment. Itzhak, Graham and Campbell (2013) study a series of psychological and economic risks that manifest at the level of management. Miles (2009) analyzes the effects of uncertainty on investments. Salman McLee (2014) are concerned with the relationship between aggregate investment and investors' feelings.

2. The correlation between risky savings and cautious demand

In the analysis above we took into consideration only the work-related risk. The individual had a chance of risk-free alternative savings but he was not sure about the size of the income that he will earn at the given time 1. considering a model in which labor income is known, but the profit rate of savings is risky. Let us consider a consumer with an investment horizon of two periods. If a lifelong income is certain, we assume without losing the generality that the entire income is paid at time $t=0$. Let us suppose that w_0 signifies wealth. The consumer’s target is:

$$\max_s V(s) \equiv u(\omega_0 - s) + \beta E u((1+\tilde{r})s)$$

The first order condition for this program is:

$$u'(\omega_0 - s) = \beta E[(1+\tilde{r})u'((1+\tilde{r})s]$$

The second-order condition is easier to identify as unfolding under the risk aversion. In fact, the objective function $V(s)$ is concave in s.

Let us consider, firstly, the case in which the savings rate without risk is r_0 as before. The first order condition in this case is:

$$u'(\omega_0 - s) = \beta(1+r_0)u'((1+r_0)s)$$

Focusing on the effects of risk once again we consider the simple case in which the expected profit rate savings equals discount rate for time preference, ie $r = \delta$ so that $=(1+r_0)^{-1}$. However, s^* satisfies $w_0 - s^* = (1+r_0)s^*$. As expected, optimal economy s^* is thus considered as there is no fluctuation in consumption between the two data: $c^*_0 = c^*_1$. We return to the question whether adding risk to profit from savings we obtain a higher level of savings, in this formula we conclude that prudence alone is not sufficient to lead to an increase in the level of savings. In fact, there are two influences that contribute to it, on the one hand, the risky nature of profit makes savings less attractive than a risk free rate with the same average profit. But, on the other hand, the term 1 of risk will induce a reason for caution to a prudent consumer. We conclude that we need a prudency level high enough to have a dominance of the reason for caution as we’ll demonstrate further in this article.

Since $V(s)$ is concave, we conclude that the uncertainty of the profit rate will force optimum level of savings whenever the folllowing relation is satisfied:

$$E[(1+\tilde{r})u'((1+\tilde{r})s)] > (1+r_0)u'((1+r_0)s)$$

This inequality is sustenable if the function $h(R)\ \ Ru'(Rs)$ is convex in R. A correct calculation shows that $h''(R)=2su''(Rs)+s^2Ru'''(Rs)$. Let us suppose that $u''<0$ and that savings are not 0, since c_1 would still be zero in this case. We conclude that $h''>0$ if:

$$\frac{-zu'''(z)}{u''(z)} > 2$$

is sustained by $z=Rs$. The left side of the equation is just a measure of the relative prudence. Thus, from the equation, we get the following property of comparative statics of risk profits by saving:

$$s^* \text{ will } \begin{cases} \text{increase} & \text{if relative prudence exceeds 2,} \\ \text{remain the same} & \text{if relative prudence equals 2,} \\ \text{decrease} & \text{if relative prudence is less than 2.} \end{cases}$$

Of course, the relative prudence should not satisfy the above mentioned conditions. However, a case in which it does, is the function of happiness of CRRA type, namely $u(c)=c^{1-\gamma}/(1-\gamma)$, where γ is the constant degree of risk aversion . We refer to "risk aversion" and not to "aversion fluctuation" because we are interested in risk and not in the consume projected in time in a risk-free construction. In this case, calculations show that relative caution is equal to γ+1. In this case, preference is obtained as follows:

$$s^* \text{ will } \begin{cases} \text{increase} & \text{if relative risk aversion exceeds 1,} \\ \text{remain the same} & \text{if relative risk aversion equals 1,} \\ \text{decrease} & \text{if relative risk aversion is less than 1.} \end{cases}$$

3. The content and significance of temporal consistency

When the model contains only two terms of consumption as above, any future action can be planned in advance at 0 term with no possibility of changing anything. At the second term, the agent only consumes what has in his savings account. When there are more than two terms, which was planned at time $t=0$ can be reviewed at time $t=1$. If you decided at $t=0$ to buy an expensive product that you decided to pay him the next term $t=1$, you still can decide at time $t=1$ to delay payment to keep your high level of consumption. Thus, consumers can have a problem of consistency over time. Let us reconsider the relation between consumption and savings under conditions of certainty described in equation (6.5), where $\Pi_t=(1+r)^{-t}$ and *n3*. At the time $t=0$, the consumers plans his consumption profile $(c_0 ,..., c_{n-1})$ for the rest of his life, which maximizes the utility of life expectancy $\sum_{t=o}^{n-1} p_t u(c_t) = \omega_0$ that is subject to budget restrictions throughout life $\sum_{t=o}^{n-1} \Pi_t c_t = \omega_0$. Let us remember that p_t is the factor used to decrease the happiness at times t in the current cycle. Using the condition for $t=1$and $t=0$, the choice is planned as follows:

$$u'(c_2) = \frac{p_1}{(1+r)p_2} u'(c_1)$$

This rule of consumption can be satisfied so as to effectively spend the money saved at time $t=0$. Anticipating how he will spend the money he saved, the agent determines his initial optimal consumption c_0. By solving the system of equations resulted in the combination with reducing the budget, the whole profile of consumption is selected as follows:

$(c_0, c_1, c_2, \dots, c_{n-1})$

Let us consider the situation generated at time $t=1$. The welfare was effective by the initial consumtion c_0, but it has also increased based on the profit r resulted from savings. At time $t=0$, the agent planned to consume c_1 at time $t=1$. However, he is ready to reconsider his choice. The welfare for the period which remained can be written as follows:

$$\sum_{t=o}^{n-1} p_{t-1} u(c_t)$$

Indexes of p parameters and c variables that are important at this moment. In particular, we should notice that the satisfaction $u(c_2)$ which occurs at a time $t=1$ is diminished in the point p_1, the diminishing factor for the horizon of a period. By maximizing the objective function and reducing the budget, we obtain:

$$\sum_{t=o}^{n-1} \Pi_{t-1} c_t = (\omega_0 - c_0)(1+r)$$

which generates the prime order condition provided by the current choice:

$$u'(c_2) = \frac{p_0}{(1+r)p_1} u'(c_1)$$

The above equations are equivalent only if $p_1/p_2=p_0/p_1$. This is the equivalent to a request for $p_t=a\beta^t$ for $t=0,1,2$, or that fall to be exponential. This terminology comes from the fact that the equivalent of the continuous time of this decreasing function is $p(t) = e^{-\delta t}$. Extending this condition for all times t implies that the fact that the choice of optimal consumption c_1 is located in $t=1$ and is not different from the the one which was planned at time $t=0$.

Conclusion

The problem is more complex when the consumer does not use the condition $p_t=a\beta^t$ for the decreasing factors. Let us suppose that p_2 is larger than the relationship p/p_0. From the initial equations there comes the conclusion that the consumption level selected c_1 at time $t=1$ is higher than the one planned at the time $t=0$. In this case, we have a problem of consistency. When determining the initial consumption, the agent can not trust himself in relation to limiting his own consumption in the future. This is typical of an addictive behavior: a consumer believes that it is better for him to consume today based on his belief that it would give up the consumption tomorrow, but when tomorrow comes, the consumer realises that it is better for him to purchase, thus postponing the tomorrow's decision to the next day, and so on. We may also suspect that such an addictive behavior can be extended to other products generating a global problem of addiction to consumption. For people who have this problem, long-term savings plans without the possibility of withdrawing money may be beneficial despite the inflexibility of these plans. The problem of temporal consistency may explain why a large number of people in developed countries find it acceptable to finance consumption based on short-term loans by using credit cards at 20% interest, and still keep money in long-term accounts with interests under 5 %.

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