SOME ELEMENTS ABOUT NORMALITY OF SINGLE-EQUATION ESTIMATORS

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Abstract

In this paper, the authors focus on the asymptotic normality of the LIML estimator, the LIML designed by Fuller and on the trend adjustment of 2-stage least squares (B2SLS). The corresponding hypotheses are presented and discussed, then the theorems for the estimators are defined.

Key words: *equation, instrument, normality, estimator, least squares*

Introduction

Anghelache and Prodan (2013) focus on the use of simple regression in studies at macroeconomic level,while Anghel and Anghelache (2015) review the application of non-linear models. Anghelache and Popovici (2015) study the significance tests, based on instrumental variables. Pagliacci, Anghelache and Mitruţ describe the utility of statistic-econometric models, as instruments of economic analysis.

In this paper we attempt to extend the results presented by Stock and Yogo (2003) by extending the results of asymptotic normality for LIML estimator (limited information maximum likelihood), FULLER's LIML modification (FLIML) and for the bias–adjusted of the smallest squares estimators in two phases (B2SLS) for the case in which the instrument weakness is in such a way that the increase rate of the concentration parameter r_n is slower than the one of the number of instruments K_n, but in such a way that $\frac{\sqrt{K_n}}{r_n} \to \infty$, for $n \to \infty$.

Thus, we will obtain results with asymptotic normality, in situations with weaker instruments than they were considered in other research.

Notations used:

Tr (·) – matrix trace

>0 – positive precission when matrix is applied

$\overline{lim}_{n\to\infty}\, a_n$–denotes the superior limit of the sequence $\{a_n\}$

Px = X (X'X)' X'–the matrix that is orthogonal projected in gama(X) and $M_x = 1 - P_x$

Model and assumptions

It is given the model with two simultaneous equations (SEM):

$$y_{1n} = y_{2n}\beta + X_n\gamma + u_{n,}$$

$$y_{2n} = Z_n\pi + X_n\varphi + v_n,$$

where, y_{1n} and y_{2n} arethe vectors n x 1 of the two endogene variables observations of the system.

X_n is an x J matrix of J exogene variables observations, included in (1) equation

Z_n is an x k_n matrix of observations upon K_n, instrumental variables or of exogene variables excluded from (2) equation.

u_n and v_n are n x 1vectors of random perturbation.

Be $\eta_i = (u_i, v_i)'$, where u_i and v_i are I component of random vectors u_n, and v_n.

The following hypotheses are made.

Hypothesis 1

$\pi=\pi_n=\frac{cn}{bn}$ for some sequences of sequential positive numbers$\{b_n\}$, un-decreasing numbers in n, and for some non random sequences k_n x 1 the vectors' parameter$\{c_n\}$.

Hypothesis 2

Be $\{\overline{z}_{i,n}$: i = 1,…,n; n≥1} a triangular set on R^{Kn+J} first range random variables, where $\overline{z}_{i,n}$= (Z' $_{i,n}$, X' $_{i,n}$)' with Z' $_{i,n}$, X' $_{i,n}$ showing i row from Z_n and X_n matrix. Supposing that

a) $K_n\to\infty$and $n\to\infty$so that$\frac{Kn}{n}\to$*á,* for an *á* constant which satisfies the condition 0≤*á*<1

b) Be m_{1n}…. . ∞ , $n\to\infty$ and we suppose that exist the following constants $\underline{D}_{ë}$and $\overline{D}_{ë}$, with $0<\underline{D}_{ë}\le\overline{D}_{ë}<\infty$, so that

$$\underline{D}_\lambda \le \underline{\lim}_{n\to\infty} \lambda_{min}\left(\frac{\overline{Z}_n'\overline{Z}_n}{m_{1n}}\right) \text{ almost certain} \quad (3)$$

and

$$\overline{\lim}_{n\to\infty} \lambda_{max}\left(\frac{\overline{Z}_n'\overline{Z}_n}{m_{1n}}\right) \le \overline{D}_\lambda \text{ almost certain} \quad (4)$$

where $\overline{Z}_n$= (Z_n, X_n)

c) There is a range of positive real numbers $\{m_{2n}\}$, undecreasingin n, and $0<\underline{D}_c \leq \overline{D}_c < \infty$, so that $\underline{D}_c \leq \underline{\lim}_{n\to\infty}(\frac{c_n' c_n}{m_{2n}})$ (5)

$$\overline{\lim}_{n\to\infty}(\frac{c_n' c_n}{m_{2n}}) \leq \overline{D}_c. \quad (6)$$

Hypothesis 3 $\bar{Z}_n$ si η_i are independent for any *i* and *n*.

Hypothesis 4

(a) $\eta_i \equiv i.i.d.(0, \Sigma)$, where $\Sigma > 0$ and $\Sigma\begin{pmatrix}\sigma_{uu} & \sigma_{vu} \\ \sigma_{vu} & \sigma_{vv}\end{pmatrix}$

(b) there is a constant D_η cu $0 < D_\eta < \infty$, when $\max\{E(u_i^8), E(v_i^8)\} \leq D_\eta$.

(c) $E(u_i^3) = E(v_i^3) = E(u_i^2 v_i) = E(u_i v_i^2) = 0$.

Hypothesis 5

We have the definition for $r_n = \frac{m_{1n} m_{2n}}{b_n^2}$. Suppose that $n \to \infty, r_n \to \infty$ such that $\frac{r_n}{K_n} \to 0$, but $\frac{\sqrt{K_n}}{r_n} \to 0$.

(i) Hypothesis 1 and 2 are similar to those of Chao and Swanson (2002).

(ii) Hypothesis 4 (c) requires a symmetry of model disturbance distribution regarding simultaneous equations given by equations (1) and (2).

(iii) Hypothesis 5 focuses on concentration parameter increasing at a slower rate than the number of instruments K_n, prevailing a more rapid rate than $\sqrt{K_n}$.

(iv) Hypotheses require a compromise with regard to the conditions relative to Donald and Newey (2001) and Stock and Yogo (2003a). Our hypothesis regarding exogenous variables are weaker than those of Donald and Newey and Stock and Yogo. On the other hand, we build more strict hypotheses with regard to the moments of error distributions.

Hypothesis 4(b) considers that the error distributions have eight finite moments, while the definite moments of Donald and Newey and Stock and Yogo have only four.

Also, hypothesis 2(a) considers a less strict condition on the increasing rate of the number of instruments confronted with the one imposed by Donald and Newey and Stock and Yogo.

The asymptotic normality of the singular equation estimators

We focus on three estimators:

1. Estimator LIML (Limited information maximum likelyhood)

$$\hat{\beta}_{LIML,n} = (y_{2n}' M_{X_n} y_{2n} - \hat{\lambda}_{LIML,n} y_{2n}' M_{\bar{Z}_n} y_{2n})^{-1} \times (y_{2n}' M_{X_n} y_{1n} - \hat{\lambda}_{LIML,n} y_{2n}' M_{\bar{Z}_n} y_{1n}) \quad (7)$$

where $\hat{\lambda}_{LIML,n}$ is the smallest root of the determinant equation.

$$\det\left\{\begin{pmatrix} y_{1n}^{'}M_{X_n}y_{1n} & y_{1n}^{'}M_{X_n}y_{2n} \\ y_{2n}^{'}M_{X_n}y_{1n} & y_{2n}^{'}M_{X_n}y_{2n} \end{pmatrix} - \lambda_n \begin{pmatrix} y_{1n}^{'}M_{\overline{Z}_n}y_{1n} & y_{1n}^{'}M_{\overline{Z}_n}y_{2n} \\ y_{2n}^{'}M_{\overline{Z}_n}y_{1n} & y_{2n}^{'}M_{\overline{Z}_n}y_{2n} \end{pmatrix}\right\} = 0 \qquad (8)$$

Estimator LIML modified Fuller FLIML

$$\hat{\beta}_{FLIML,n} = (y_{2n}^{'}M_{X_n}y_{2n} - \hat{k}_{FLIML,n}y_{2n}^{'}M_{\overline{Z}_n}y_{2n})^{-1} \times (y_{2n}^{'}M_{X_n}y_{1n} - \hat{k}_{FLIML,n}y_{2n}^{'}M_{\overline{Z}_n}y_{1n}) \qquad (9)$$

where $\hat{k}_{FLIML,n} = \hat{\lambda}_{LIML,n} - \frac{a}{n-K_n-J}$, for a positive constant *a*.

Estimator B2SLS (Bias – corrected two stage least-squares)

$$\hat{\beta}_{B2SLS,n} = (y_{2n}^{'}M_{X_n}y_{2n} - \left(\frac{n}{n-K_n+2}\right) y_{2n}^{'}M_{\overline{Z}_n}y_{2n})^{-1} \times \left(y_{2n}^{'}M_{X_n}y_{1n} - \left(\frac{n}{n-K_n+2}\right) y_{2n}^{'}M_{\overline{Z}_n}y_{1n}\right) \qquad (10)$$

All these three estimators are special cases of an estimator of k class defined by:

$$\hat{\beta}_{k,n} = (y_{2n}^{'}M_{X_n}y_{2n} - ky_{2n}^{'}M_{\overline{Z}_n}y_{2n})^{-1}(y_{2n}^{'}M_{X_n}y_{1n} - ky_{2n}^{'}M_{\overline{Z}_n}y_{1n}) \qquad (11)$$

The following theorems present the main asymptotic results of this paper:

Theorem 1 (LIML)
Let $\hat{\beta}_{LIML,n}$ be defined as in equation 7. Under hypotheses 1 –5, we have

$$\left(\frac{\psi_n}{\sigma_{L,n}}\right)(\hat{\beta}_{LIML,n} - \beta_0) \xrightarrow{d} N(0,1), n \to \infty, \text{unde } \psi_n = b_n^{-2}c_n^{'}Z_n^{'}M_{X_n}Z_nc_n$$

Theorem 2 (FLIML) Let $\hat{\beta}_{FLIML,n}$ be defined as in equation 9. Under hypotheses 1 – 5, we have

$$\left(\frac{\psi_n}{\sigma_{L,n}}\right)(\hat{\beta}_{FLIML,n} - \beta_0) \xrightarrow{d} N(0,1), n \to \infty,$$

Theorem 3 (B2SLS) Let $\hat{\beta}_{B2SLS,n}$ be defined as in equation 10. Under hypotheses 1 –5, we have

$$\left(\frac{\psi_n}{\sigma_{L,n}}\right)(\hat{\beta}_{B2SLS,n} - \beta_0) \xrightarrow{d} N(0,1), n \to \infty,$$

Theorem 4 Suppose that hypotheses 1 - 5 are performed and $\eta_i \sim E_2(0, \Theta)$, where $\Theta = \tau\sum$ for a positive constant ô and E_2(0, è). Then, there is a positive whole number N such that $n \geq N$.

$$\sigma_{B,n}^2 > \sigma_{L,n}^2$$

Conclusions

We derived the limits of the distributions of estimators LIML and B2SLS by configurating some weak instruments for which convergence parameter is supposed to increase at a weaker rate than the number of K_n instruments, but at a more rapid rate than $\sqrt{K_n}$.

As a conclusion, we obtained normal asymptotic results of these estimators with regard to weaker instruments than in the other papers that used a weaker instruments framework.

Regarding our paper, both rates, the convergent and the variance one, they are different from the cases with strong instruments, which are cases when the instruments increase at the same or bigger rate than K_n.

Also, we found that estimator B2SLS is not asymptotic equivalent with LIML and FLIML under weaker instruments use.

References

1. Anghelache C., Prodan L. (2013) – „*The Use of Simple Regression in Macroeconomic Analysis*”, Knowledge Horizons – Economics, Volume (Year): 5 (2013), Issue (Month): 4 (December), pp. 168-172
2. Anghelache C., Popovici M. (2015) – „*Theoretical Elements Regarding the Tests of Significance Based on Instrumental Variables*”, Romanian Statistical Review Supplement, Volume (Year): 63 (2015), Issue (Month): 9 (September), pp. 82-90
3. Anghelache C., Anghel M.G. (2015) – „*Main aspects regarding some non-linear models used in economic analyses*”, Romanian Statistical Review Supplement, Volume (Year): 63 (2015), Issue (Month): 9 (September), pp. 7-10
    1. Donald S.G., Newey W.K. (2001) – „*Choosing the number of instruments*”, Econometrica, Vol. 69, No. 5 (Sep., 2001), pp. 1161-1191
4. Pagliacci M., Anghelache C., Mitruţ C. (2014) – „*Economic Analysis through the Use of Statistical - Econometric Models*”, Romanian Statistical Review Supplement, Volume (Year): 62 (2014), Issue (Month): 4 (April), pp. 32-43
5. Stock J.H., Yogo M. (2003) – „Asymptotic Distributions of Instrumental Variables Statistics with Many weak Instruments”, working paper, Harvard University